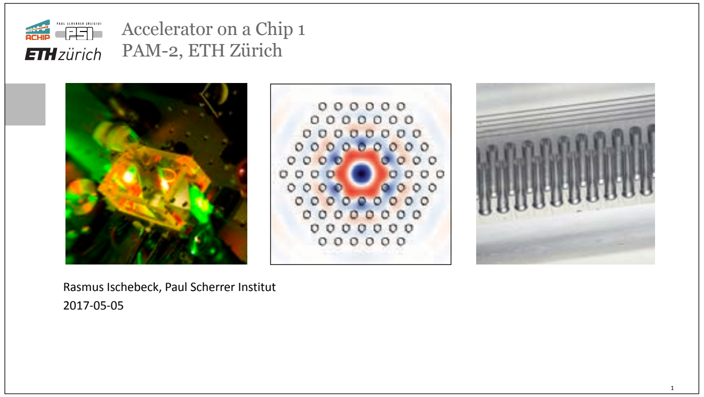
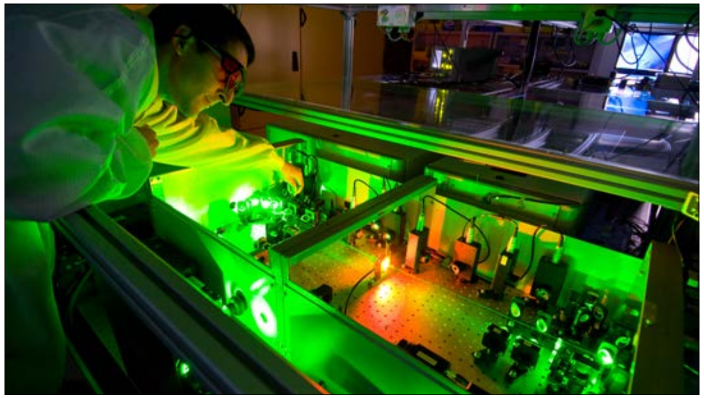


Accelerators on a Chip

Imagine a sunny day in Zurich.
About 500 trillion photons from the sun reach Zurich every femtosecond.
Now imagine that you could take all these photons, and align them coherently in a micrometer-size cavity.
You would get enormous electromagnetic fields, which you could use to accelerate charged particles.



Such power densities can actually be achieved by femtosecond lasers!
People have been proposing to build laser-based accelerators since a long time.

Laser-Based Accelerators

Shimoda
Appl. Opt. **1 (1)**, 33 (1961)

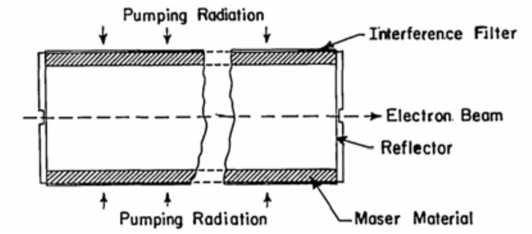


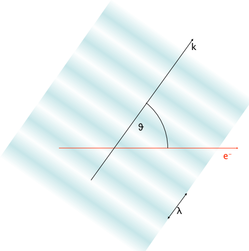
Fig. 1. Schematic diagram of an electron linear accelerator by optical maser.

In fact, the idea to accelerate electrons with a laser is older than the word "laser"!

How to Accelerate Charged Particles

Assume:

- an ultrarelativistic particle of charge e
- moving along the z axis
- accelerated by a plane electromagnetic wave that propagates at an angle ϑ to the z axis



But one needs more than an electromagnetic wave to accelerate!

Let’s look what happens in a plane wave

How to Accelerate Charged Particles

Then:

- Position of the electron

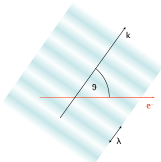
$$\vec{r}(t) = \begin{pmatrix} 0 \\ 0 \\ ct \end{pmatrix}$$

- Electric field

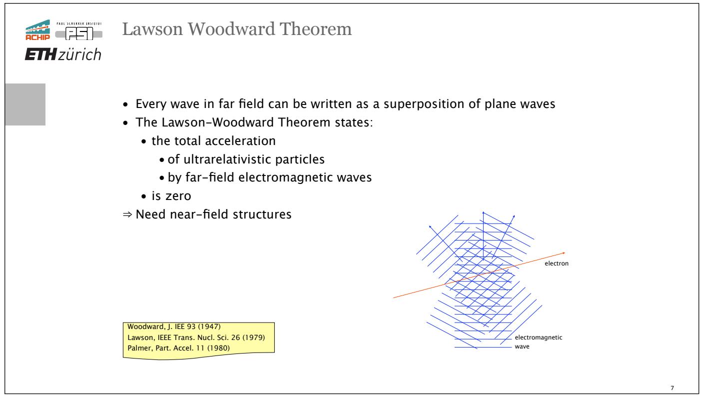
$$E_\parallel = \sin\vartheta \cos\left(\omega t - \frac{z}{2\pi\lambda\cos\vartheta}\right)$$

- Energy gradient

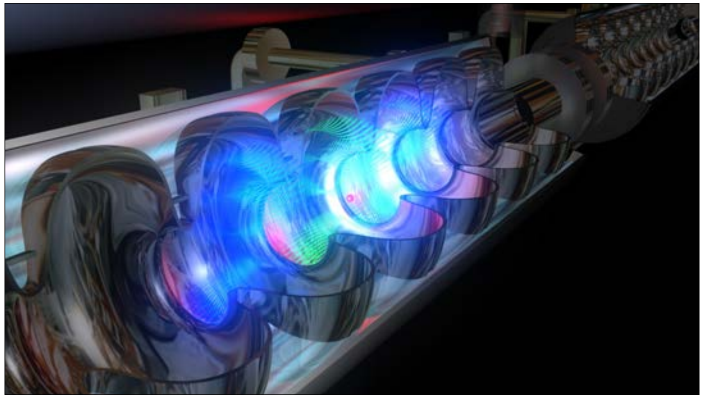
$$\frac{\Delta W}{L} = \frac{\int_L eE_\parallel dz}{L} = \frac{\int_L \sin\vartheta\cos(kz(1-\sec\vartheta))dz}{L} = \frac{\sin\vartheta\sin(kL(1-\sec\vartheta))\frac{1}{k(1-\sec\vartheta)}}{L} \xrightarrow{L\to\infty} 0$$



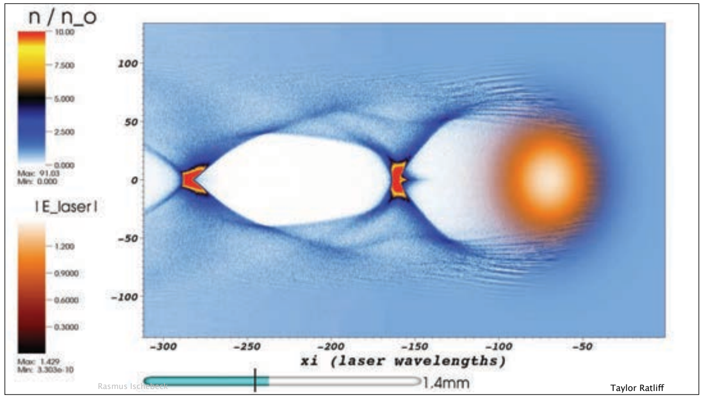
The average acceleration by plane waves in vacuum is zero…



—> Requirement of near-field structures



No acceleration is possible with plane waves in vacuum.
Need to provide boundary conditions to Maxwell’s equations
—> Need for an accelerating structure
This accelerating structure is at a distance comparable to the wavelength of the fields.
Here: superconducting cavity for 1.3 GHz fields, corresponding to 23 cm wavelength in vacuum.
> What can we do to get structures with a size suitable for laser acceleration?



Plasma waves have a longitudinal electric field
Here: plasma generated by an ultrashort laser pulse
Phase velocity is equal to the speed of light
Electrons can be captured in the wave

Or: build accelerating structures from dielectrics
Using manufacturing techniques of the semiconductor industry and advanced free-form manufacturing techniques

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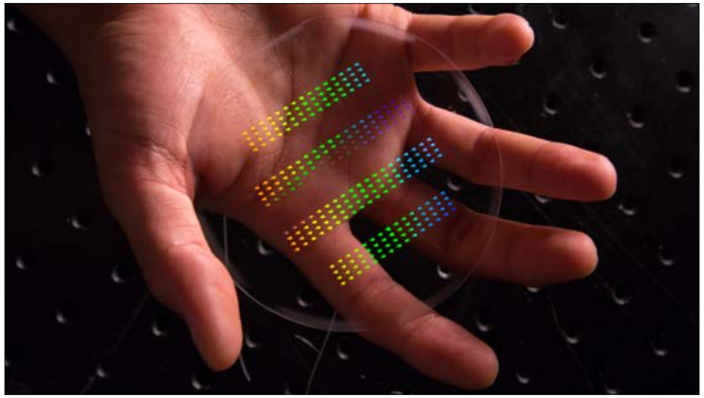
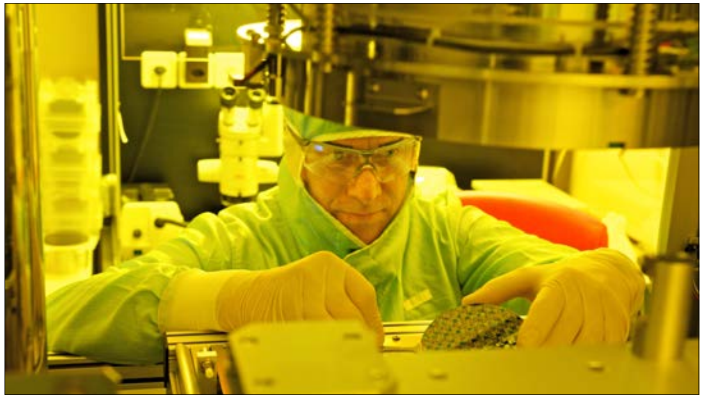


Structures have micrometer size
Hundreds of structures fit on the palm of a hand



Challenges: structure manufacturing
Semiconductor industry is investing 3.2e11 CHF/y (3 SwissFELs / day) into manufacturing of micrometer-scale structures
Different advanced manufacturing techniques
Now supplemented by free-form methods

Requirements on an Accelerating Structure

- Longitudinal electric field
- Phase velocity = particle velocity
- Uniform fields across the particle bunch
  - Low energy spread
  - Good emittance
- Sustain large fields
  - Fields on the surface should be comparable to the fields seen by the beam

---

Rectangular Waveguide

- Starting with Maxwell's Equations for vacuum:

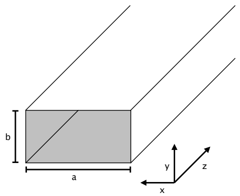
$$\nabla \cdot \vec{E} = 0$$
$$\nabla \cdot \vec{H} = 0$$
$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$
$$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

- Ansatz for the electromagnetic fields:

$$\vec{E}(x,y,z,t) = \tilde{E}(x,y) \cdot e^{i(\omega t - kz)}$$
$$\vec{H}(x,y,z,t) = \tilde{H}(x,y) \cdot e^{i(\omega t - kz)}$$

- Decomposing into longitudinal and transverse components

$$\tilde{E}(x,y) = E_T(x,y) + \vec{e}_z E_z(x,y)$$
$$\tilde{H}(x,y) = H_T(x,y) + \vec{e}_z H_z(x,y)$$



Assume a rectangular waveguide made from perfect conductors
Solve Maxwell's equations to find waves that can propagate inside the waveguide
—> look for modes with a wave-like behavior in z

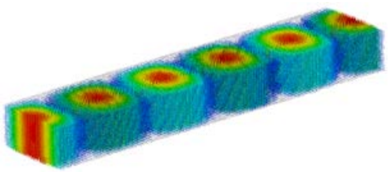
Rectangular Waveguide

- Solving the wave equation
- Take into account the boundary conditions
- Transverse electric modes

$$H_z(x,y) = H_0 \sin\left(\frac{\pi m}{a} x\right) \sin\left(\frac{\pi n}{b} y\right)$$
$$E_z(x,y) = 0$$
$$m, n \in \mathbb{N}$$

- Transverse magnetic modes

$$E_z(x,y) = E_0 \sin\left(\frac{\pi m}{a} x\right) \sin\left(\frac{\pi n}{b} y\right)$$
$$H_z(x,y) = 0$$



E-field phase animation inside a rectangular waveguide.
Source: cst.com

Solving these equations would exceed the scope of this lecture… I will give only the most important results!

Transverse electric and transverse magnetic modes
Good news: TM modes have longitudinal electric fields
(Unlike in vacuum, where TEM modes have transverse electric (and magnetic) fields)

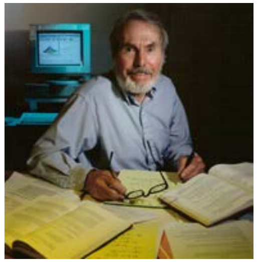
Rectangular Waveguide

- A few important points:
  - Transverse fields can be calculated from this
  - There is a cutoff frequency
  - Waves with lower frequency cannot propagate in the waveguide

$$f_c = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

  - Phase velocity of the mode

$$v_p = \frac{\omega}{k} = \frac{c}{\sqrt{1-\frac{f_c^2}{f^2}}} > c$$



More details: see Jackson, Electrodynamics

Fields propagate above a certain minimum frequency
Phase velocity is always larger than the speed of the particles

Rectangular waveguides can thus not be used as particle accelerators!

Circular Waveguides

- Similar results for circular waveguides
- Field distribution is given by the Bessel functions

$$\boldsymbol{E}_t = \frac{ik}{\gamma^2}\nabla_\perp\psi$$

$$\boldsymbol{B}_t = \frac{\mu_r\epsilon_r\omega}{ck}\boldsymbol{e}_z \times \boldsymbol{E}_\perp$$

$$\psi_{mn}(r,\phi) = J_m(u_{mn}r/b)e^{im\phi}, \quad m = 0,1,2,\ldots, \quad n = 1,2,\ldots$$

- Again, the phase velocity is always larger than the particle velocity

$$v_p > c$$

The situation is similar for circular waveguides

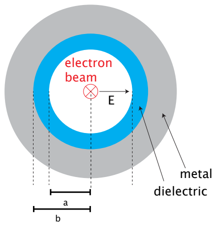
Dielectric Lined Waveguide

- Assuming a geometry with:

$$\epsilon_r(r) = \begin{cases} 1, & 0 \le r \le a \\ \epsilon_r, & a \le r \le b \end{cases}$$

- Continuity of the electromagnetic fields at the boundaries:

$$E_z(r = b) = 0$$
$$E_z(r_+ = a) = E_z(r_- = a)$$
$$B_\phi(r_+ = a) = B_\phi(r_- = a)$$
$$\epsilon_0\epsilon_r E_r(r_+ = a) = \epsilon_0 E_r(r_- = a)$$



Modify this structure by adding a dielectric layer with some dielectric constant inside the metallic waveguide

Dielectric Lined Waveguide

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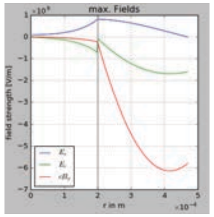
- Fields can be obtained analytically

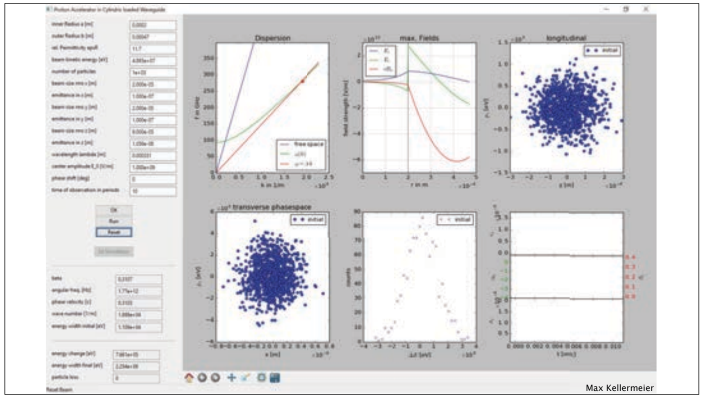
$$E_z = \begin{cases} E_1 J_0(\gamma_1 r) e^{i(kz-\omega t)}, & r < a \\ E_2 \left(J_0(\gamma_2 r) - \frac{J_0(\gamma_2 b)}{Y_0(\gamma_2 b)} Y_0(\gamma_2 r) \right) e^{i(kz-\omega t)}, & a < r < b \end{cases}$$

$$E_r = \begin{cases} i\frac{k}{\gamma_1} E_1 J_0'(\gamma_1 r) e^{i(kz-\omega t)}, & r < a \\ i\frac{k}{\gamma_2} E_2 \left(J_0'(\gamma_2 r) - \frac{J_0(\gamma_2 b)}{Y_0(\gamma_2 b)} Y_0'(\gamma_2 r) \right) e^{i(kz-\omega t)}, & a < r < b \end{cases}$$

$$B_\phi = \begin{cases} i\frac{\omega}{\gamma_1 c^2} E_1 J_0'(\gamma_1 r) e^{i(kz-\omega t)}, & r < a \\ i\frac{\omega}{\gamma_2 c^2} \epsilon_r E_2 \left(J_0'(\gamma_2 r) - \frac{J_0(\gamma_2 b)}{Y_0(\gamma_2 b)} Y_0'(\gamma_2 r) \right) e^{i(kz-\omega t)}, & a < r < b \end{cases}$$



Analytic expressions for the electromagnetic fields can be found



Tool to calculate the fields, and to propagate a particle beam through this structure
Written by Max Kellermeier as a semester thesis
See link at the end of this talk

Experiments

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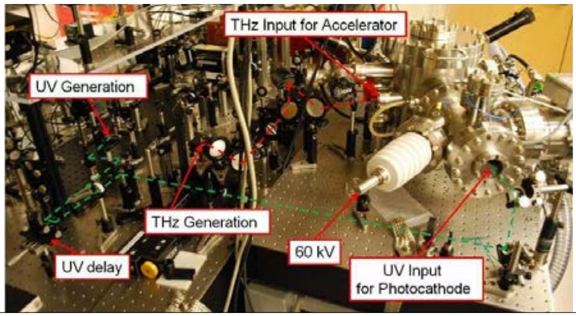


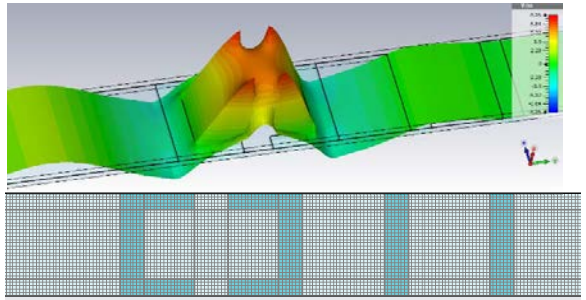
Experiments have been performed at MIT with an accelerator with this geometry
Results will be shown here next week

Accelerating Structures

- Design goals
  - Optimize the overlap of short-pulse laser with the electron bunch
  - Adapt to changing velocity of the particles
  - Coupling of field into the structure
  - Maximize the ratio between accelerating field and surface field
- Numerical modeling of the structures
  - Eigenmodes of the structures
  - Acceleration of single particles
  - Collective effects
  - Tolerance studies

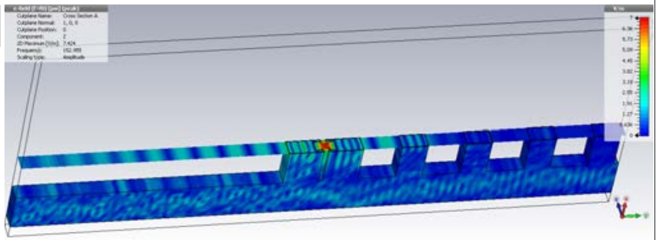
Typically, the electromagnetic fields can not be expressed by analytical formulae
—> numerical modeling

Time Domain Simulations

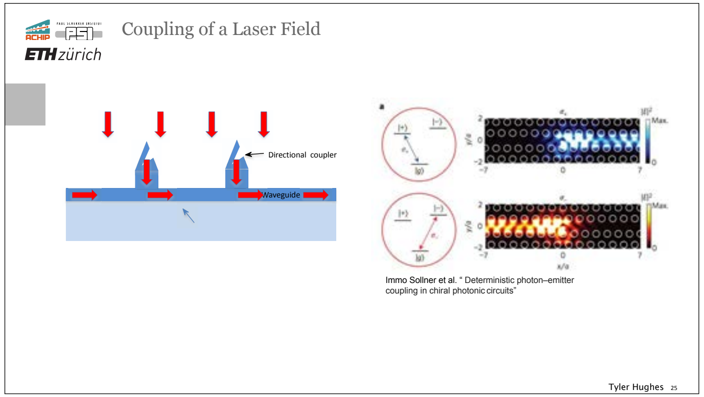


Structure shown on the bottom, using vacuum (white) and dielectric (cyan)
Electric fields shown in the top plot
Periodic structure
Periodicity in the simulation achieved by boundary conditions, such that only one cell needs to be modeled

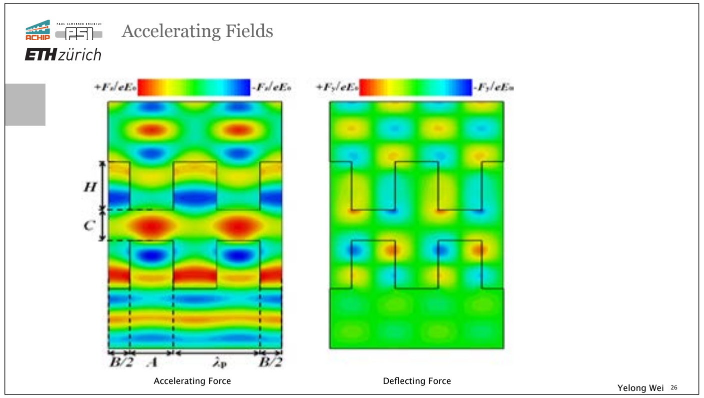
Simulation of the Electromagnetic Field in 3D



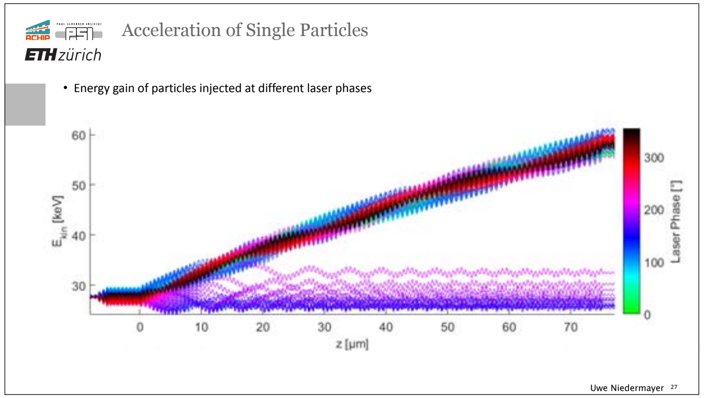
Of course, the simulation calculates the electromagnetic field on a three-dimensional grid



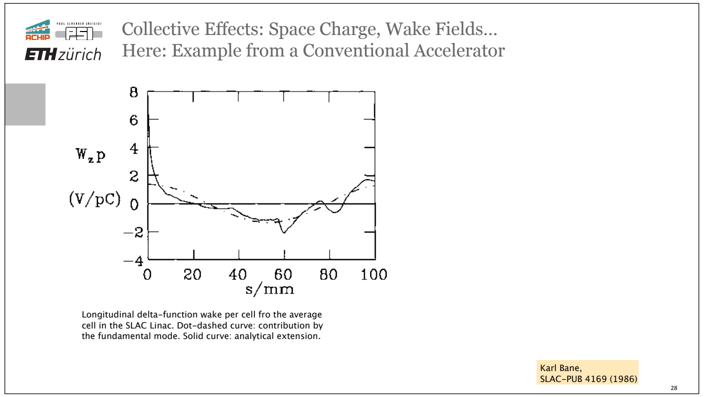
Coupling of an external field into the structure
Here: coupling of a plane wave in vacuum to the waveguide



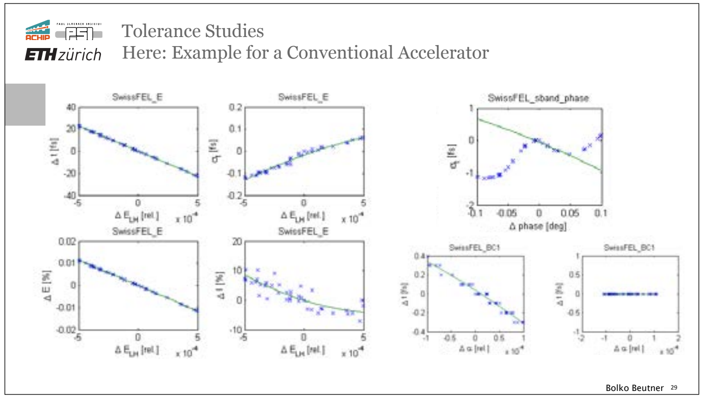
Accelerating and deflecting forces can be calculated from the electromagnetic fields



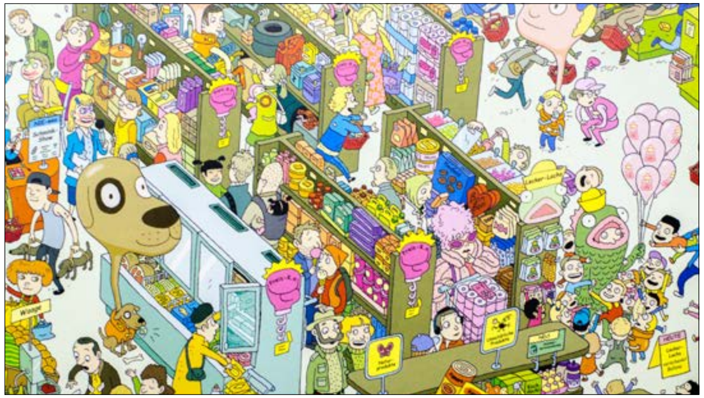
Energy gain is calculated by tracking single particles along the structure



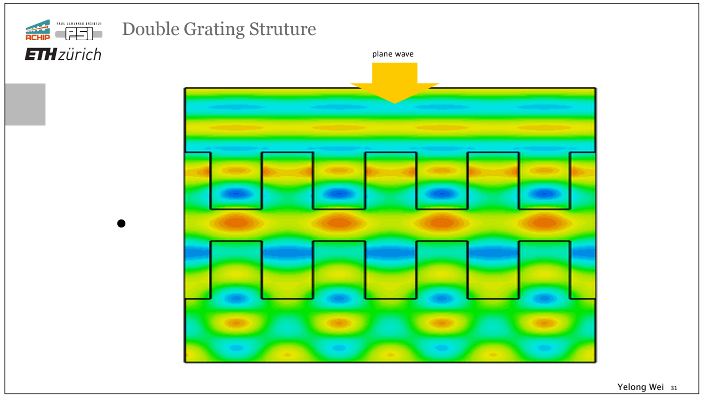
Next steps in the simulation are presently being carried out by my colleagues
Results are not yet presentable, thus I am showing here calculations performed for classical accelerators (metallic structures, 3 GHz)



Tolerance studies important to build accelerators that work more than once…

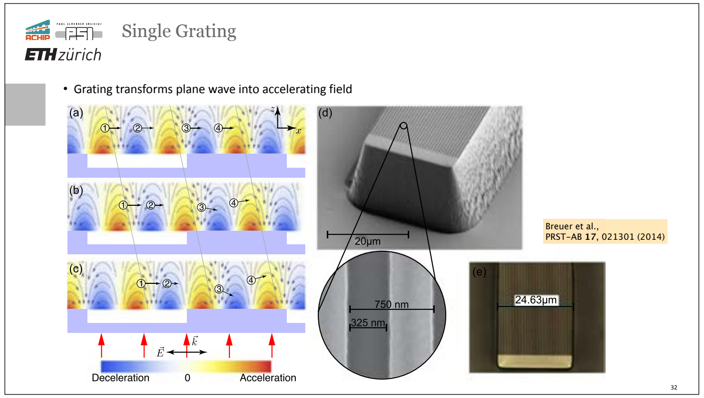


A large variety of accelerating structures has been invented…



Among the first structures that were investigated experimentally:
dielectric lined circular waveguides, and gratings

Grating transforms a plane wave into an accelerating mode by periodically re-phasing the field



Double grating is difficult to produce and to align
—> initial experiments performed with single grating

Symmetry is lost, and fields are less uniform. But: acceleration has been clearly demonstrated!

(shown here: third harmonic of the grating)

Double grating is difficult to align.
Maybe some inspiration came when a researcher was walking around Stanford campus…

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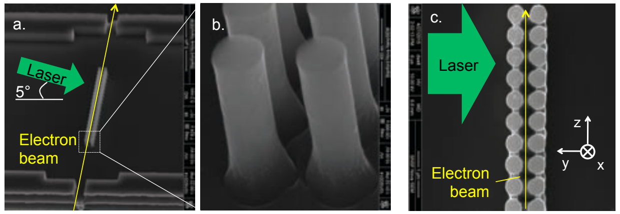
Dual Pillar Structure



Use vertical columns instead of the grating
Alignment done during fabrication process

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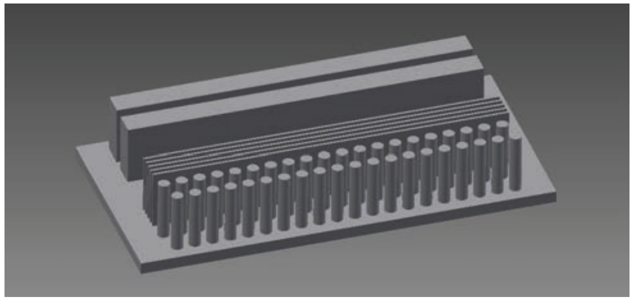
Double-Pillar Structure with Bragg Reflector



Add a series of walls behind the columns to symmetrize the fields even further

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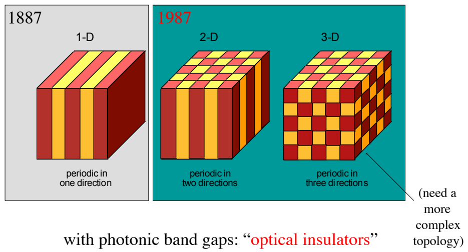
Towards 3-Dimensional Structures: Photonic Crystals

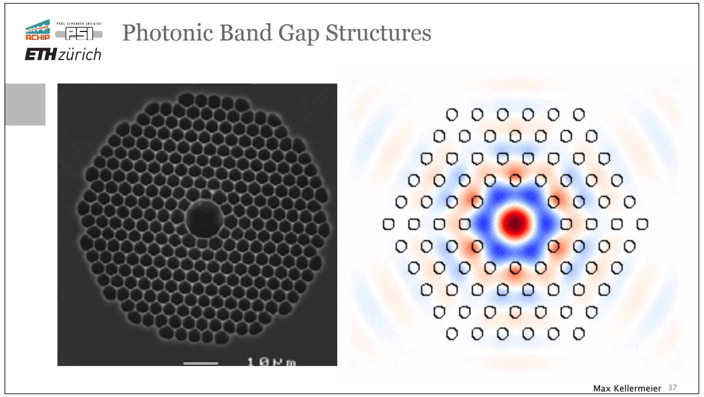
periodic electromagnetic media



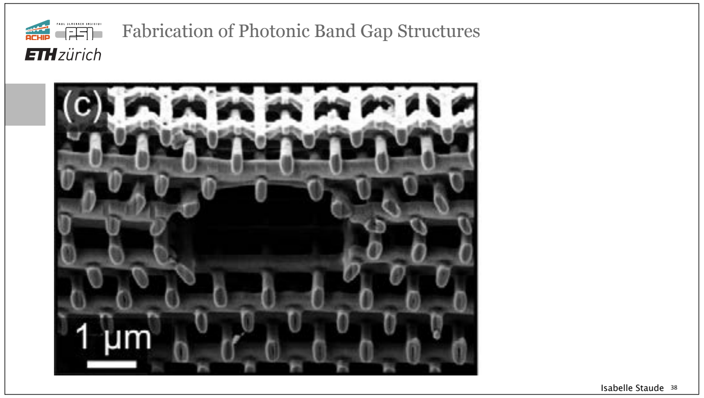
with photonic band gaps: "optical insulators"

All dielectric structures presented so far were resonators with very low Q.
Possibility to improve on this: contain the electromagnetic fields.

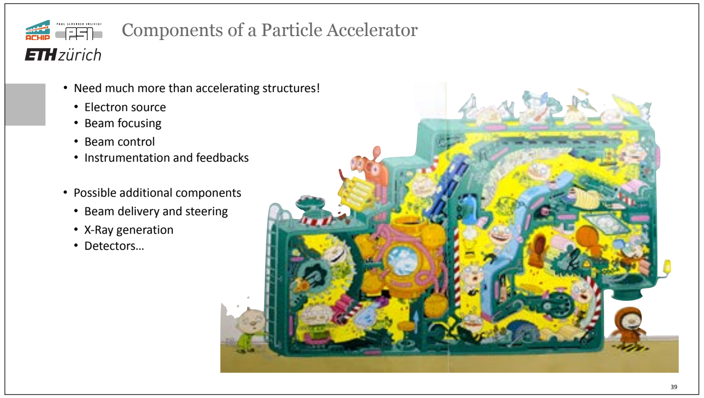
Metals support relatively low field amplitudes.
Maybe photonic band gap structures can be used?



Photonic band gap structures are widely used in fiber optics
Holes contain TE waves well (microscope image, left), rods contain TM waves (simulation, right)

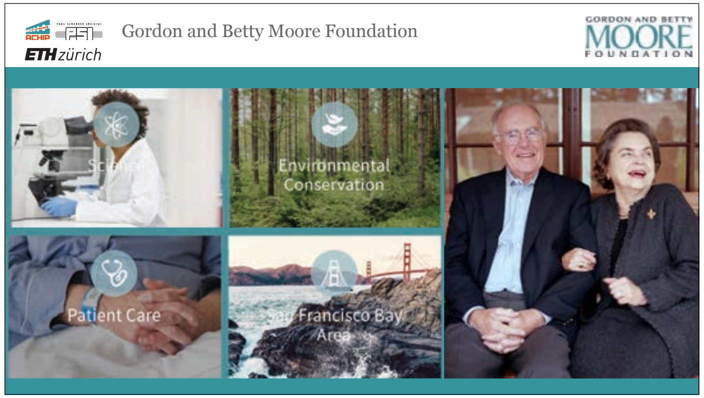


Three-dimensional photonic band gap structure fabricated at KIT



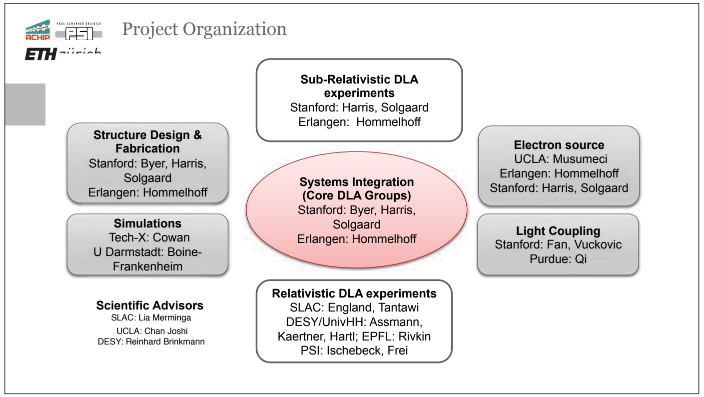
All of these components will have to be built for our accelerator-on-a-chip!

International effort to pursue this research
Funding by the Gordon and Betty Moore Foundation

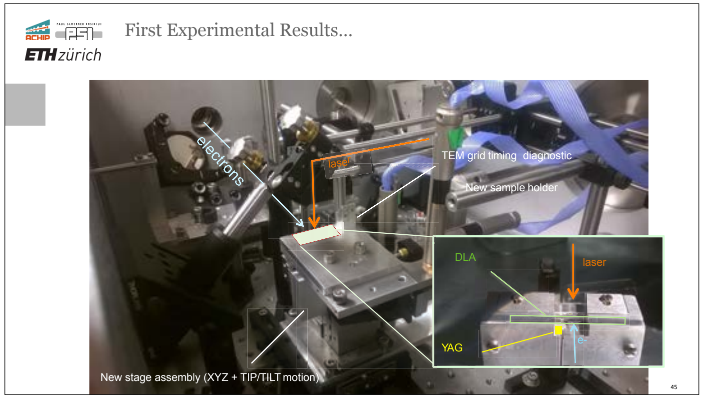


Foundation that funds basic research

Collaboration started in 2015



Collaboration has grown significantly in the last 18 months



First experimental results by this collaboration will be presented next week

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Thank You

- Thank you for Simulations, Measurements, Illustrations, Animations and Photos:
  - Ainu Havukainen
  - CST
  - Franz Kärtner
  - Gordon and Betty Moore Foundation
  - Google
  - Ken Leedle
  - Marie Siegler
  - Max Kellermeier
  - Nick Veasey
  - Peter Hommelhoff
  - Paul Scherrer Institut
  - Stanford University
  - Steven Johnson
  - Tyler Hughes
  - Uwe Niedermayer
  - Yelong Wei

- Download this talk in the file format of your choice:
- https://ischebeck.net

- Max' software to model dielectric lined waveguide accelerators:
- https://github.com/scimax/PyPACHIP

- More information on the Accelerator-on-a-Chip International Program at PSI:
- http://achip.ch